t(minutes)	0	4	9	15	20
W(t) degrees F	55.0	57.1	61.8	67.9	71.0

2012 #1

The temperature of water in a tub at time t is modeled by a strictly increasing, twice differentiable function, W, where W(t) is measured in degrees Fahrenheit and t is measured in minutes. At time t = 0, the temperature of the water is 55° F. The water is heated for 30 minutes, beginning at time t = 0. Values of W(t) at selected times t for the first 20 minutes are given in the table above.

c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)dt$. Use a left Riemann sum with four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\frac{1}{20}\int_0^{20} W(t)dt = \frac{1}{20}\left[(4-0)(55)+(9-4)(57.1)+(15-9)(61.8)+(20-15)(67.9)\right]$$

Underestimate b/c W(t) is strictly increasing